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THE CONCEPT OF PSEUDO-STANDARDIZED VARIABLES AND ITS USE AS ELEMENTS OF SHAPE OPERATORS

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FOREWORD

The research work reported herein was conducted by Prof. Dr. Waloddi Weibull, Chemin Fontanettaz 15, 1012 Lasusanne, Switzerland under USAF Contract No. F44620-72-C-0028. This contract, which was initiated under Project No. 7351, "Metallic Materials", Task 735106, "Behavior of Metals", was administered by the European Office, Office of Aerospace Research. The work was monitored by the Metals and Ceramics Division, Air Force Materials Laboratory, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio, under the direction of Mr. W. J. Trapp, AFML/LL.

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This technical report has been reviewed and is approved.

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ABSTRACT

The concept of pseudo-standardized variable is explained and the fundamental properties of this variable are indicated. Its most important property of being scale and location invariant makes it useful as elements of shape operators, and its space being equal to the closed interval (0,1) has practical advantages.

Four types of shape operators are defined and examined. Twenty-five tables which simplify their practical applications have been prepared and are presented. Two examples concerning data of rotating beam fatigue performance illustrate the different numerical procedures.

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INTRODUCTION

If the distribution function of the random variable x can be put in the form

$$P = F[(x - \mu)/\beta] \tag{1}$$

then the variable

$$z = (x - \mu)/\beta \tag{2}$$

is called the corresponding standardized variable. Introducing (2) into (1) its distribution function becomes

$$P = F(z) \tag{3}$$

from which it immediately follows that z is both scale and location invariant, that is, the order statistics z_i are, for any given sample size N, uniquely determined by the function F which may or may not involve a shape parameter α.

In cases where the parameters β and μ are unknown it is possible to substitute for them the estimates $\hat{\beta}$ and $\hat{\mu}$, resulting in a random variable

$$z' = (x - \hat{\mu})/\hat{\beta} \tag{4}$$

On the condition that these estimates are unbiased, also the variable z' is both scale and location invariant.

A much simpler procedure consists in substituting x_1 for $\hat{\mu}$ and $(x_N - x_1)$ for $\hat{\beta}$, where x_1 is the smallest and x_N the largest of the order statistics of the sample $[x_1]$. We thus arrive at the random variable

$$t = (x - x_1)/(x_N - x_1) \tag{5}$$

which will be called <u>the pseudo-standardized variable</u>. It has been found to possess some useful properties which will now be demonstrated.

II

FUNDAMENTAL PROPERTIES OF THE PSEUDO-STANDARDIZED VARIABLE

The most important property of the variable t is its scale and location invariance, which may be proved by introducing x from equ.(2) into (5). Then we have

$$t = (z - z_1)/(z_N - z_1) \tag{6}$$

which proves the double invariance, since all z_i are uniquely determined by the distribution function F.

Any given sample $[x_i]$ is easily transformed into a sample $[t_i]$ by use of equ.(5). From this formula it follows that

$$t_1 = 0 \quad ; \quad t_N = 1 \tag{7}$$

and the number of non-fixed elements of $[t_i]$ is thus reduced to $(N-2)$.

It is a convenient property for the writing of computer programs that all the order statistics t_i are finite and belong to the interval $(0,1)$, which simplifies the dividing of the t-space into classes.

Any operator acting upon a sample $[t_i]$ constitues a shape operator, due to its invariances. Four types of operators have been thoroughly examined as will be indicated in the following. Each of them may be used for testing normality of the sample and also for testing the acceptability of any other assumed distribution function.

It should be pointed out that any such test has its individual capacity of disclosing special types of deviations of the given sample from the set of samples belonging to the assumed hypothetical sample. It has been observed that shape operators based on pseudo-standardized samples are well fitted for disclosing large outliers due to the fact that such outliers will appear as the largest order statistic x_N. From equ.(5) it is easily concluded that a too large x_N will depress all the other statistics t_i below their normal values. This behaviour is of particular importance for the analysis of fatigue-test data, where large outliers are frequently observed within small samples for the reason that the sample may emanate from a two-component population.

For the practical use of these tests, numerical tables are required. So far, twenty-five such tables have been prepared and are appended. They may also contribute to a statistical description of the pseudo-standardized variable.

III

FOUR TYPES OF SHAPE OPERATORS

Each single order statistic t_i may be used as a test statistic for deciding between two different assumed distribution functions or for testing the acceptability of single distributions. Even if the decision power will, in this way, be very small, it may, in many cases, be sufficient for rejecting an assumed function. (Cf.3.3)

It is a priori evident that a combination of a part or all of the order statistics t_i will greatly improve the decision power. Four such possibilities will now be demonstrated.

3.1 The operator T(j,k,N)

The most simple procedure of combining the order statistics consists in summing all or some of them. This sum will be denoted by

$$T(j,k,N) = \sum_{i=j}^{k} t_i/(k-j+1) \qquad (8)$$

It may be considered an operator. If applied to a given sample $[t_i]$ it yields a test value. Let the hypothetical population be denoted by R. We now have to generate a large number of random samples from this population, transform them into $[t_i]$ samples and let the operator $T(j,k,N)$ act upon them. The result is a large number of values which may be considered to be random points of the test statistic, which is denoted by $TR(j,k,N)$. All those values which are equal to the test value are counted. If the number is zero or small, the hypothetical distribution function will be rejected.

So far, this procedure has been applied to the Weibull distribution with $\alpha = 1.0$, 0.9, 0.7, 0.5, 0.3, 0.1, 0.01. These populations are symbolized by $R = \alpha$. It has also been applied to the normal distribution, which is symbolized by

$R = 0$. Thus $T1(j,k,N)$ and $T0(j,k,N)$ signify the operator $T(j,k,N)$ acting upon exponential and normal random samples, respectively.

An improvement is obtained by using two different sets of (j,k) and combining them to a bivariate test statistic. This procedure has been performed by use of an IBM 360, M75 and Program 1/71, which produces the frequency matrix, the test level distribution, and the decision power of several combinations. From these tables test-level matrices can be prepared. Some examples are presented in Table 1 - 6.

These matrices are very useful. Each of them provides, in fact, three different tests of acceptability, as will be demonstrated by an example. Suppose that a sample $[x_i]$ of size $N = 10$ is given, and that its normality will be tested. After transformation into a $[t_i]$ sample two test values are computed and found to be, say, as indicated below. From Table 3 the corresponding values of Q are obtained

$T(2,9,10) = 0.575$ $Q = 26.1\%$
$T(2,5,10) = 0.475$ $Q = 11.8\%$
Bivariate $(.575,.475)$ $TL = 17.8\%$

In this table $Q = 1 - P$ signifies the chance of finding a single value larger than the test value. Since as well too small as too large test values are undesirable, the rejection regions of the univariate statistics will be defined by $Q > 97.5\%$ and $Q < 2.5\%$, while that of the bivariate statistic by $TL < 5\%$ for a 5% level of significance. From the tables it is found that these rejection regions do not coincide. In view of the fact that the bivariate test statistic provides more information, the rejection will be based on its test levels.

3.2 The operator ST(j,k,N)

Another way of combining the order statistics t_i consists in summing the squared deviations from the expected values $\bar{t}_i$. The test operator thus becomes

$$ST(j,k,N) = \sum_{i=j}^{k} (t_i - \bar{t}_i)^2 \tag{9}$$

The expected values $\bar{t}_i$ may be arbitrarily specified.

If they correspond to the population R, the operator will be denoted by $TR(j,k,N)$, in particular the notations $T1(j,k,N)$ and $TO(j,k,N)$ indicate that the expected values correspond to the exponential and the normal distributions, respectively. These operators, if acting upon random samples drawn from the population S constitute test statistics, which will be denoted by $STRS(j,k,N)$. The decision powers are obtained by comparing the sampling dbns of STRR and STRS. The test statistics of acceptability tests are of the type $STRR(j,k,N)$.

Also in this case, the sample may be split up into two or more parts, thus forming bi- or trivariate statistics.

Expected values $\bar{t}_i$ and their variances can be computed by use of the Program 8/71, which produces also the sampling dbns of the order statistics t_i.

Some expected values t_i and their variances are listed in Tables 7 - 11, and a test level matrix of a bivariate test statistic in Table 12, which may be used for testing normality. In this case, the test value should be as small as possible. For this reason the rejection regions will be defined by $Q < 5\%$ and $TL < 5\%$ for a 5% level of significance. Graphs of $\bar{t}_i$ are shown in Fig.1 and 2. If the test values t_i are plotted in this graph, a preliminary selection of the distribution function can be made.

3.3 The operator TI

This operator consists in computing the percentiles t_P of the orders 5% and 95% and using them as the limits of the rejection regions. The criterion of rejection says that the hypothetical distribution function is rejected, if anyone of the elements of the sample $[t_i]$ is larger than the corresponding percentile $t_{.95}$ or smaller than $t_{.05}$ as listed in the tables.

Some percentiles are listed in Tables 13 - 16 and sampling dbns in Table 17 - 19, from which other percentiles can be determined.

Graphs of the percentiles for various sample sizes and $\alpha = 1.0;0.01$ and O(normal dbn) are shown in Figs 3 and 4. If anyone of the order statistics t_i lies above the curve

for $\alpha = 0.01$ or below that for $\alpha = 1.0$ it can be concluded that the sample does belong neither to the normal nor to any Weibull dbn, which may be taken as an indication that the sample is inhomogeneous and may belong to a two-component population, which frequently occurs, when fatigue-life distributions are concerned. Some rejection regions are given in Fig.5.

3.4 The operator VJ

This operator consists in dividing the space of the pseudo-standardized variable t into r classes without common points and counting the number v_i of elements of the sample $[t_i]$ which fall within each of the classes. The test value is defined by

$$VJ = (v_1, \ldots, v_r)$$

where the class frequencies v_i may be regarded as the coordinates in the r-dimensional space of the test statistic VJ. A more detailed description and necessary tables are presented in an earlier publication. (Cf.Ref.[1].)

IV
NUMERICAL EXAMPLES

The examples are taken from a very large collection of groups of fatigue-performance data compiled at the Boeing Company by Whittaker & Besuner [2].

The first example is taken from a technical note by Hardrath, Utley & Guthrie [3] on rotating beam fatigue tests of notched and unnotched 7075-T6 aluminum alloy specimens. The test data x_i, fatigue life in kilocycles, are listed in Table 24. This sample will be tested for normality and log-normality.

It is first transformed into a t-sample after which it is practical to start with the TI-test. Comparing the order statistics t_i with the 5% percentiles in Table 14 for $\alpha = 0$ we find that all t_i fall below the rejection limits. Consequently, the hypothesis that the sample belongs to a normal population is strongly rejected. Furthermore, the values of t_i also fall below the rejection limit corresponding to $\alpha = 1.0$. Hence, it can be concluded that the sample does not belong to any Weibull population. We now take the

logarithms of x_i and transform them into a new set of t_i and repeat the procedure. Also the hypothesis of log-normality is strongly rejected.

If the test data x_i are examined it becomes evident that the rejections are caused by the very large outlier $x_{10} = 3,318$ kc which may indicate that the sample, in this case, is drawn from a two-component population and that the outlier belongs to the second component.

The values of TX(2,5,10) and TX(2,9,10), defined by equ.8, and STOX(2,5,10) and STOX(2,9,10), defined by equ.9, are now computed. The corresponding values of Q and TL can be read from Tables 3 and 12. Both normality and log-normality are rejected also by all the tests.

Finally, the test value VJX is determined by use of the class limits $t_c = 0;0.250;0.375;0.500;1.0$ and the test levels TL are taken from Table 7 in Ref.[1]. Both hypotheses are strongly rejected.

<u>The second example</u> is also taken from the Boeing Collection. The procedure indicated above is repeated, this time with a positive result, as demonstrated in Table 25.

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3. Hardrath, H.F., Utley, E.C., and Guthrie, D.E. "Rotating beam fatigue tests of notched and unnotched 7075-T6 aluminum alloy specimens under stresses of constant and varying amplitudes." NASA TN D-210, Dec. 1959.

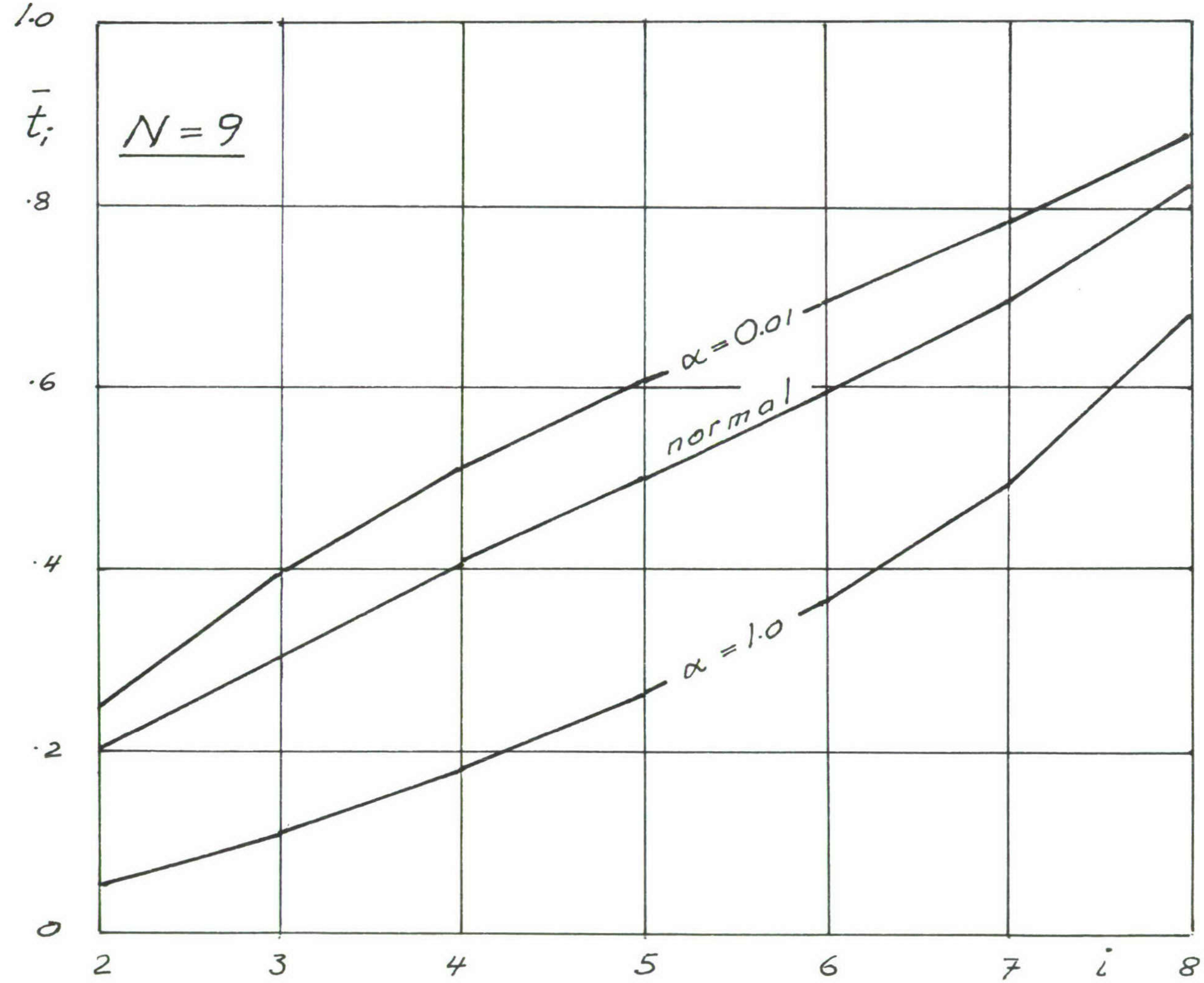

Fig. 1 Expected values $\bar{t}_i$ for $N = 9$; $\alpha = 1.0$; 0.01; 0.

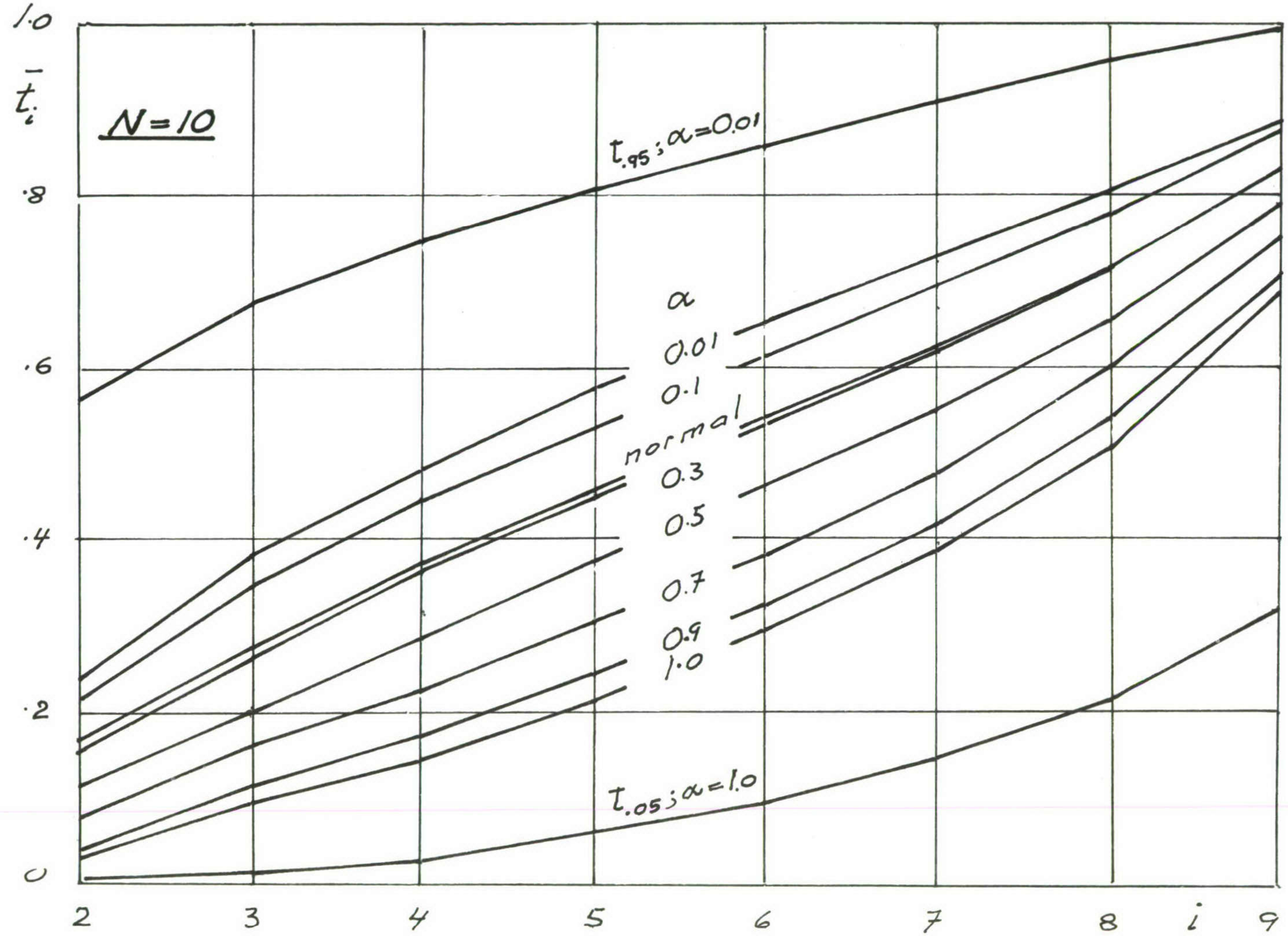

Fig. 2 Expected values $\bar{t}_i$ for $N=10$; $\alpha=1.0; 0.9; 0.7; 0.5; 0.3; 0.1; 0.01$ and normal dbn. Percentiles for $P=5\%$; $\alpha=1.0$ and $P=95\%$; $\alpha=0.01$

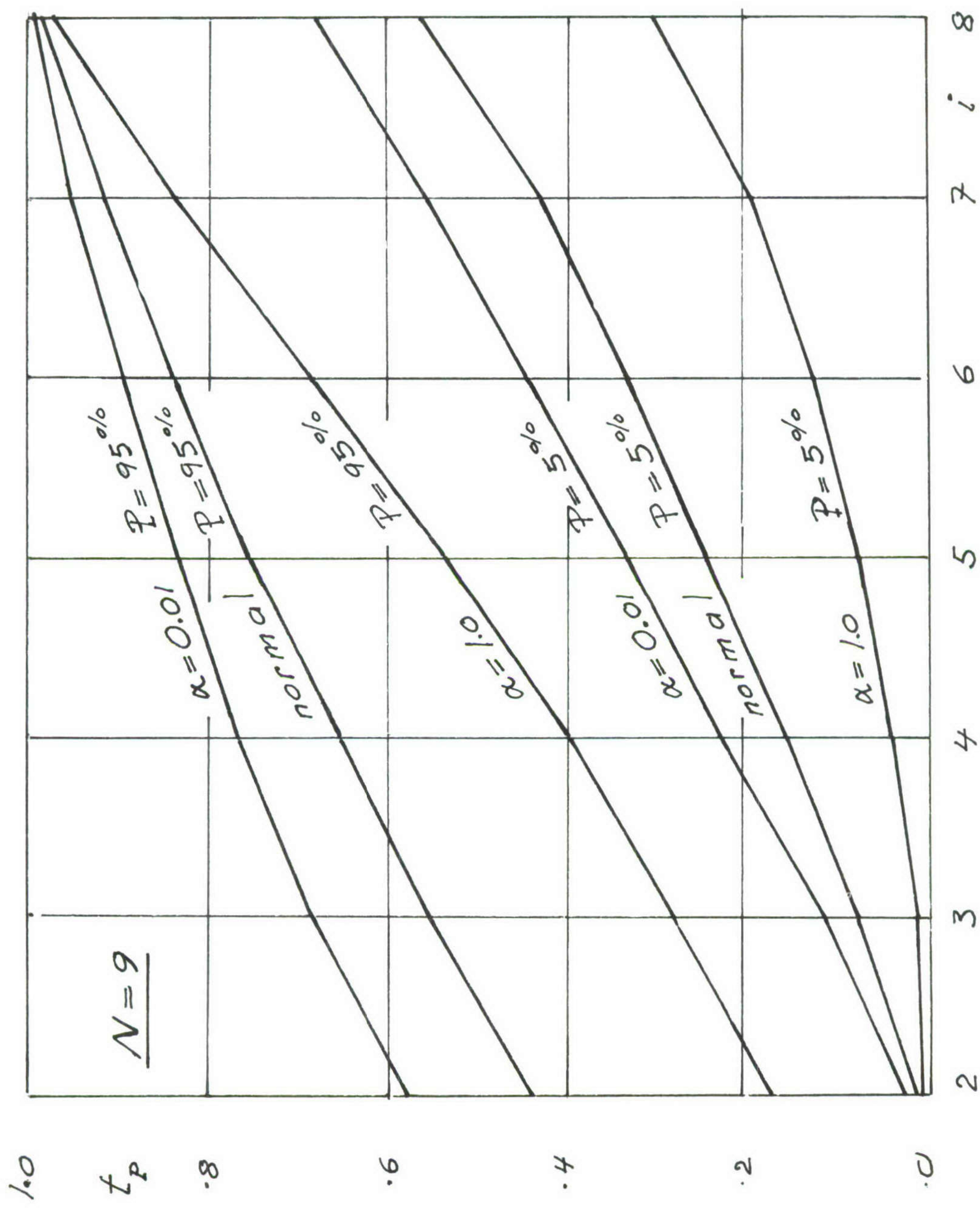

Fig. 3. Percentiles t_P of orders 5% and 95%
$N = 9$; $\alpha = 1.0$; 0.01 and normal dbn.

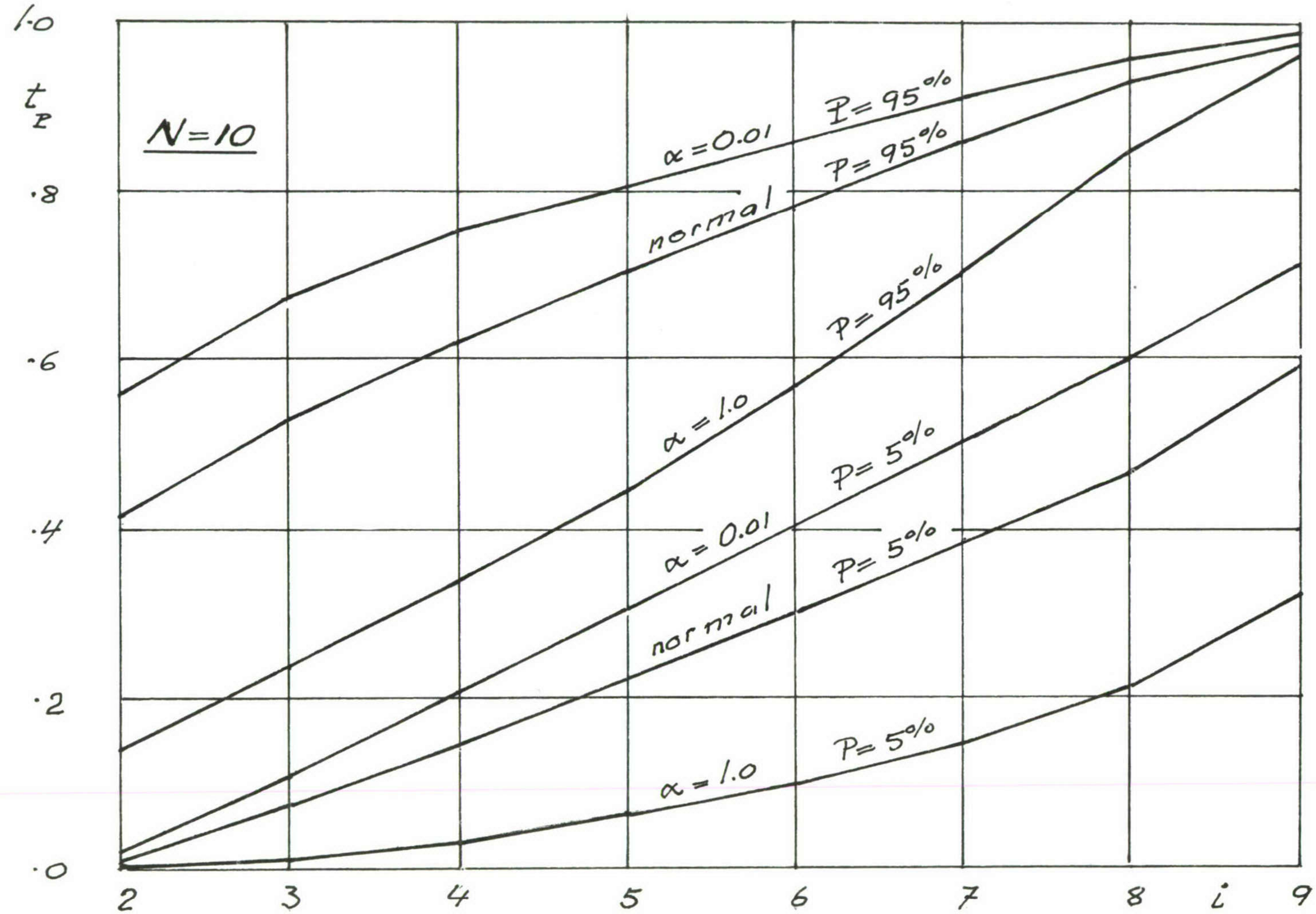

Fig. 4. Percentiles t_P of orders 5% and 95%
$N=10$; $\alpha = 1.0$; 0.01 and normal dbn.

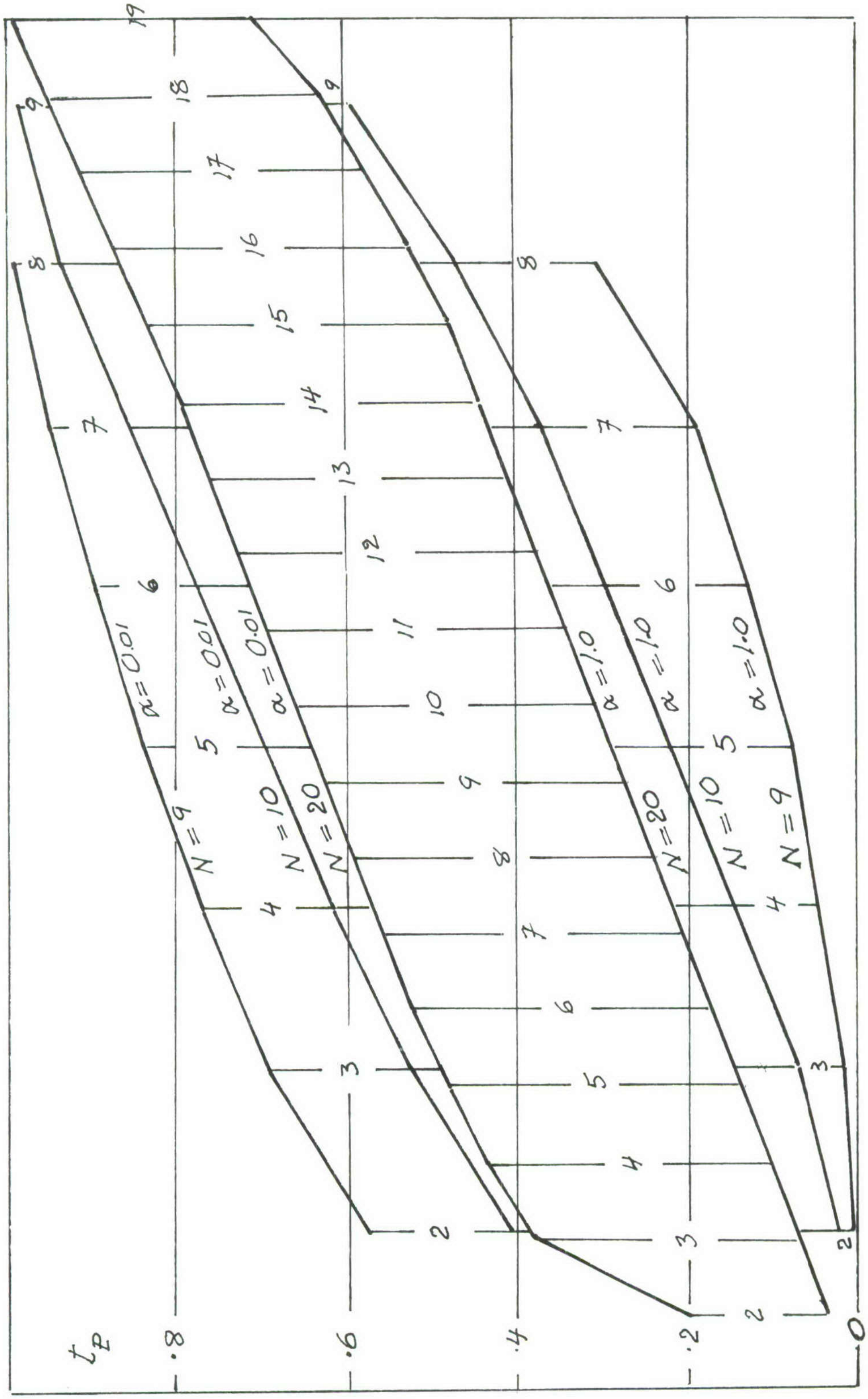

Fig. 5 Rejection regions of the operator TI for $N = 9, 10$ and 20

Table I. Test level matrix of the statistic [T1(2,9,10) + T1(2,5,10)]

$T_1 \backslash T_2$	.025	.050	.075	.100	.125	.150	.175	.200	.225	Q_1
.075	5.6	8.2	-	-	-	-	-	-	-	99.6
.100	13.2	27.3	3.2	-	-	-	-	-	-	98.4
.125	8.7	48.7	25.3	-	-	-	-	-	-	96.3
.150	10.7	63.4	43.8	13.5	-	-	-	-	-	92.9
.175	11.0	55.7	74.4	31.9	6.8	-	-	-	-	88.2
.200	7.6	52.8	92.1	68.5	24.7	-	-	-	-	81.7
.225	4.8	42.6	97.2	85.0	45.0	14.6	3.7	-	-	73.8
.250	3.2	33.6	78.5	100.0	72.4	33.6	11.6	-	-	65.0
.275	-	25.9	66.7	94.6	87.3	51.4	21.2	3.7	-	55.9
.300	-	11.6	41.4	80.6	89.6	63.4	36.2	16.3	4.5	46.8
.325	-	7.2	24.7	60.1	82.8	78.5	57.2	30.3	6.8	37.8
.350	-	4.5	16.3	34.4	55.7	70.5	65.0	30.3	16.3	29.9
.376	-	-	12.2	22.3	39.2	58.6	51.4	40.3	19.6	23.1
.400	-	-	3.7	12.2	21.2	38.2	47.4	46.2	35.3	17.0
.425	-	-	-	3.2	12.8	22.3	31.1	37.2	28.8	12.4
.450	-	-	-	2.4	4.1	14.2	17.6	28.0	26.6	8.7
.475	-	-	-	-	-	2.1	10.2	19.6	18.1	5.7
.500	-	-	-	-	-	-	4.8	7.2	10.2	3.8
.525	-	-	-	-	-	-	-	2.8	3.7	2.4
Q_2	98.2	88.3	73.0	56.9	42.6	30.9	21.7	14.7	10.3	-

$T_1 \backslash T_2$	.250	.275	.300	.325	.350	.375	.400	.425	Q_1
.325	2.1	-	-	-	-	-	-	-	37.8
.350	8.2	-	-	-	-	-	-	-	29.9
.375	14.2	2.4	-	-	-	-	-	-	23.1
.400	17.6	6.8	2.8	-	-	-	-	-	17.0
.425	22.9	12.8	5.1	-	-	-	-	-	12.4
.450	24.7	17.6	5.1	2.1	-	-	-	-	8.7
.475	19.6	20.1	10.2	6.1	2.1	-	-	-	5.7
.500	9.4	15.0	10.7	8.2	4.5	-	-	-	3.8
.525	8.9	9.4	8.7	7.6	4.1	2.4	-	-	2.4
.550	-	6.8	5.6	6.1	4.1	3.7	-	-	1.5
.575	-	2.8	-	5.6	6.1	3.2	2.1	-	0.8
.600	-	-	-	-	-	2.8	2.1	2.1	0.4
Q_2	6.9	4.4	3.0	2.0	1.2	0.7	0.5	0.3	-

Empty classes indicate a TL <2%

Table II, Test level matrix of the statistic [T(.01)(2,9,10) ++ T(.01)(2,5,10)]

$T_1 \backslash T_2$	.10	.15	.20	.25	.30	.35	.40	.45	Q_1
.30	2.0	2.5	-	-	-	-	-	-	99.4
.35	1.3	5.2	9.5	1.6	-	-	-	-	98.0
.40	1.4	7.2	15.4	13.2	4.7	-	-	-	94.8
.45	-	3.9	17.8	41.1	12.4	6.1	-	-	88.0
.50	-	1.1	10.1	33.4	60.0	35.9	12.3	-	77.8
.55	-	-	-	11.5	49.6	73.3	56.3	14.3	64.6
.60	-	-	-	-	10.8	52.7	88.6	83.4	48.9
.65	-	-	-	-	-	4.3	16.5	78.3	33.6
.70	-	-	-	-	-	-	-	16.5	19.1
Q_2	99.3	97.5	94.7	86.7	77.1	65.9	54.1	41.8	-

$T_1 \backslash T_2$	.50	.55	.60	.65	.70	.75	.80	.85	Q_1
.60	20.6	1.1	-	-	-	-	-	-	48.9
.65	94.0	28.7	1.8	-	-	-	-	-	33.6
.70	64.1	100.0	49.6	3.5	-	-	-	-	19.1
.75	2.8	24.4	68.6	43.9	5.6	-	-	-	9.0
.80	-	-	6.7	31.0	38.4	8.9	-	-	3.1
.85	-	-	-	-	8.3	19.2	8.3	-	0.6
.90	-	-	-	-	-	-	3.2	2.3	0.0
Q_2	30.4	20.2	12.1	6.6	3.1	1.2	0.3	0.0	-

Empty classes indicate a TL $<1\%$

Table III. Test level matrix of [TO(2,9,10) + TO(2,5,10)]

$T_1 \backslash T_2$	.075	.100	.125	.150	.175	.200	.225	.250	.275	.300	.325	.350	.375	Q_1
.200	3.0	-	-	-	-	-	-	-	-	-	-	-	-	99.8
.225	2.5	3.0	2.5	-	-	-	-	-	-	-	-	-	-	99.5
.250	4.0	6.0	6.4	-	-	-	-	-	-	-	-	-	-	99.0
.275	-	7.9	11.3	8.5	6.0	-	-	-	-	-	-	-	-	98.1
.300	4.0	7.6	15.8	18.9	16.8	14.9	-	-	-	-	-	-	-	96.5
.325	-	6.9	16.4	27.6	26.0	24.0	12.6	4.0	-	-	-	-	-	94.0
.350	-	4.4	15.8	25.5	34.3	46.3	29.3	12.6	2.5	-	-	-	-	90.7
.375	-	4.4	15.8	20.9	40.4	56.8	38.3	34.3	14.9	4.0	-	-	-	86.4
.400	-	-	8.5	21.8	48.7	70.0	67.8	47.9	27.0	17.5	2.5	-	-	80.9
.425	-	-	6.4	14.9	28.7	52.1	65.4	80.7	61.0	31.1	16.4	-	-	74.4
.450	-	-	-	7.6	19.7	49.3	71.4	87.7	92.1	63.2	37.0	12.6	6.0	66.6
.475	-	-	-	4.0	11.3	19.7	50.3	54.9	90.6	96.7	67.8	40.4	17.1	58.6
.500	-	-	-	-	-	9.9	25.5	53.0	80.7	100.0	98.3	75.3	46.3	49.6
.525	-	-	-	-	-	-	9.5	25.5	41.9	86.2	80.7	95.1	70.2	41.5
.550	-	-	-	-	-	-	4.9	10.4	26.5	30.5	64.3	90.6	93.6	33.7
.575	-	-	-	-	-	-	-	-	9.5	20.9	32.4	59.9	83.4	26.1
.600	-	-	-	-	-	-	-	-	-	5.3	11.3	34.9	56.8	19.5
.625	-	-	-	-	-	-	-	-	-	-	4.4	6.0	23.1	14.1
.650	-	-	-	-	-	-	-	-	-	-	-	-	6.9	9.4
Q_2	99.4	98.2	96.1	93.0	88.6	82.5	75.7	68.4	60.7	52.6	45.3	38.1	31.2	-

$T_1 \backslash T_2$	.400	.425	.450	.475	.500	.525	.550	.575	.600	.625	.650	.675	.700	Q_1
.475	7.9	-	-	-	-	-	-	-	-	-	-	-	-	58.6
.500	23.1	4.0	-	-	-	-	-	-	-	-	-	-	-	49.6
.525	44.1	14.0	4.9	-	-	-	-	-	-	-	-	-	-	41.5
.550	62.0	47.9	11.6	4.9	-	-	-	-	-	-	-	-	-	33.7
.575	86.2	76.6	42.6	17.8	3.2	-	-	-	-	-	-	-	-	26.1
.600	82.0	72.7	52.1	34.3	14.0	4.9	-	-	-	-	-	-	-	19.5
.625	44.1	57.8	74.0	38.3	37.0	9.5	5.3	-	-	-	-	-	-	14.1
.650	11.3	23.1	40.4	58.9	53.9	46.3	12.6	6.0	-	-	-	-	-	9.4
.675	-	2.5	13.2	24.0	35.6	41.9	30.5	13.2	2.5	-	-	-	-	6.2
.700	-	-	3.0	9.9	18.9	28.1	32.4	21.3	7.6	-	-	-	-	3.6
.725	-	-	-	-	2.5	9.5	18.9	20.1	14.0	8.5	4.0	-	-	2.0
.750	-	-	-	-	-	-	3.2	5.3	10.1	9.5	6.4	2.5	-	1.1
.775	-	-	-	-	-	-	-	-	3.0	6.9	7.6	3.0	-	0.5
.800	-	-	-	-	-	-	-	-	-	-	-	3.0	4.4	0.2
Q_2	24.9	19.6	15.3	11.8	8.7	6.1	3.0	2.6	1.8	1.2	0.7	0.4	0.2	-

Empty classes indicate a TL < 2 %

Table IV. Test level matrix of [T1(2,10,20)+ T1(11,19,20)]

$T_1 \backslash T_2$	.150	.175	.200	.225	.250	.275	.300	.325	.350	.375	.400	.425	.450	Q_1
.025	3.5	3.5	3.5	4.3	-	-	-	-	-	-	-	-	-	99.2
.050	6.3	13.7	29.1	28.2	52.1	34.3	40.1	46.3	31.1	26.6	25.8	10.5	12.5	88.2
.075	-	5.0	10.3	11.3	43.8	68.1	86.0	94.1	96.9	76.5	81.2	72.2	55.3	64.6
.100	-	-	-	4.2	7.4	16.9	40.1	55.3	83.5	78.8	88.6	100.0	91.3	39.9
.125	-	-	-	-	-	3.5	8.6	15.0	19.0	37.7	52.1	60.6	70.1	22.3
.150	-	-	-	-	-	-	-	-	5.9	8.6	15.0	20.8	34.3	11.3
.175	-	-	-	-	-	-	-	-	-	-	4.2	5.9	9.4	5.3
Q_2	99.6	99.0	97.6	96.2	93.2	89.6	84.4	78.2	71.2	64.4	56.7	48.6	40.6	-

$T_1 \backslash T_2$	.475	.500	.525	.550	.575	.600	.625	.650	.675	.700	.725	750	Q_1	
.050	5.9	3.5	3.5	-	-	-	-	-	-	-	-	-	88.2	
.075	46.3	28.2	19.0	10.9	7.4	-	-	-	-	-	-	-	64.6	
.100	64.2	62.4	60.6	41.3	16.9	15.0	6.3	5.9	-	-	-	-	39.9	
.125	74.3	66.1	60.6	37.7	31.1	24.2	13.7	17.9	-	-	-	-	22.3	
.150	34.3	47.7	49.1	37.7	43.8	23.5	19.5	17.9	4.2	7.4	-	-	11.3	
.175	16.9	15.5	23.5	25.8	21.4	20.1	11.7	22.5	10.3	3.5	5.9	-	5.3	
.200	2.1	7.4	8.6	12.5	10.9	13.7	8.8	8.6	5.0	5.0	4.2	3.5	2.4	
.225	-	-	-	9.4	-	7.4	8.6	4.2	3.5	-	4.2	-	1.1	
.250	-	-	-	3.5	-	-	-	3.5	-	3.5	-	-	0.6	
.275	-	-	-	-	-	-	-	-	-	-	-	3.5	0.2	
Q_2	33.5	26.8	20.2	14.8	10.8	7.5	5.4	3.0	1.9	1.0	0.4	0.1	-	

Empty classes indicate a TL < 2 %

Table V. Test level matrix of [T(.01)(2,10,20) + T(.01)(11,19,20)]

$T_1 \backslash T_2$	.600	.650	.700	.750	.800	.850	.900	.950	Q_1	
.200	-	3.2	-	-	-	-	-	-	98.9	
.250	4.2	5.6	10.2	7.7	3.2	-	-	-	96.5	
.300	5.1	15.6	23.7	20.4	16.7	2.2	-	-	90.7	
.350	-	11.8	29.9	53.0	40.0	15.6	2.4	-	80.0	
.400	-	6.6	25.7	68.1	77.4	37.2	8.9	-	65.4	
.450	-	-	15.6	49.5	88.1	72.7	19.0	-	49.7	
.500	-	-	4.2	32.0	94.0	100.0	34.5	3.2	32.6	
.550	-	-	-	6.6	64.1	82.0	43.0	3.5	19.7	
.600	-	-	-	-	17.7	56.7	60.4	8.3	10.6	
.650	-	-	-	-	-	23.7	46.1	12.7	4.7	
.700	-	-	-	-	-	7.7	29.9	10.9	1.3	
.750	-	-	-	-	-	-	4.6	9.5	0.2	
Q_2	98.3	95.0	87.1	71.6	46.4	20.7	3.8	0.0	-	

Empty classes indicate a TL < 2 %

Table VI. Test level matrix of [TO(2,10,20) ++ TO(11,19,20)]

T_1 \ T_2	.400	.425	.450	.475	.500	.525	.550	.575	.600	.625	.650	Q_1
.125	-	4.5	-	3.0	-	4.5	3.0	-	-	3.0	-	99.4
.150	-	4.5	3.0	5.5	7.4	8.7	6.4	8.7	7.4	3.0	4.5	98.1
.175	4.5	4.5	5.5	8.7	4.5	14.0	12.3	12.3	12.3	12.3	6.4	95.9
.200	-	3.0	3.0	9.4	12.9	26.3	36.6	28.8	36.0	36.0	22.2	91.4
.225	-	-	6.4	9.4	14.0	15.6	36.0	51.7	40.7	40.7	28.8	85.6
.250	-	-	-	4.5	14.0	23.5	26.3	48.3	61.1	61.1	65.1	77.3
.275	-	-	-	-	6.4	21.4	39.3	45.1	66.2	75.7	84.8	67.5
.300	-	-	-	-	-	6.4	22.2	30.3	58.1	79.5	90.4	57.4
.325	-	-	-	-	-	-	8.7	19.8	42.8	54.4	91.9	46.3
.350	-	-	-	-	-	-	-	10.0	17.9	37.2	67.3	36.8
.375	-	-	-	-	-	-	-	-	14.0	26.8	36.0	27.9
.400	-	-	-	-	-	-	-	-	-	9.4	28.8	20.5
.425	-	-	-	-	-	-	-	-	-	-	7.4	14.3
.450	-	-	-	-	-	-	-	-	-	-	7.4	8.8
.475	-	-	-	-	-	-	-	-	-	-	3.0	5.5
Q_2	99.8	99.5	99.1	98.2	96.9	94.6	91.2	86.6	80.5	73.1	64.1	-

T_1 \ T_2	.675	.700	.725	.750	.775	.800	.825	.850	.875	.900	Q_1
.150	-	3.0	-	-	-	-	-	-	-	-	98.1
.175	5.5	5.5	3.0	-	-	-	-	-	-	-	95.9
.200	15.6	10.9	3.0	3.0	-	-	-	-	-	-	91.4
.225	36.0	23.5	17.9	6.4	4.5	-	-	-	-	-	85.6
.250	64.1	42.8	28.8	26.3	12.3	-	4.5	-	-	-	77.3
.275	89.0	51.7	61.1	30.3	10.9	5.5	4.5	-	-	-	67.5
.300	94.9	82.1	63.1	39.3	19.4	19.4	3.0	-	-	-	57.4
.325	100.0	96.4	82.1	79.5	42.8	17.9	5.5	5.5	3.0	-	46.3
.350	89.0	98.1	94.9	54.4	56.2	26.3	10.9	-	3.0	-	36.8
.375	73.2	68.5	89.0	83.5	73.2	43.6	15.6	4.5	3.0	-	27.9
.400	36.0	52.6	69.6	73.2	75.7	51.7	17.9	17.9	6.4	-	20.5
.425	26.3	31.4	55.3	77.0	63.1	48.3	30.3	19.4	7.4	-	14.3
.450	12.3	26.3	46.7	46.7	58.1	51.7	44.3	17.9	3.0	-	8.8
.475	3.0	9.4	15.6	21.4	37.9	31.4	28.8	17.9	8.7	4.5	5.5
.500	-	3.0	8.7	12.3	21.4	23.5	32.5	21.4	8.7	-	3.0
.525	-	-	-	4.5	14.3	10.0	12.9	10.9	4.5	-	1.8
.550	-	-	-	-	5.5	7.4	8.7	6.4	7.4	3.0	1.1
.575	-	-	-	-	-	-	4.5	10.0	5.5	4.5	0.5
.600	-	-	-	-	-	-	5.5	4.5	3.0	-	0.2
Q_2	53.3	43.8	32.6	23.2	14.7	8.7	4.6	1.9	0.6	0.1	-

Empty classes indicate a TL < 2 %

Table VII. Expected values and variances of the pseudo-standardized order statistics t_i; $N = 9$; $\alpha = 1.0$; 0.01; 0.

i	$\alpha = 1.0$		$\alpha = 0.01$		$\alpha = 0$	
	$\bar{t}_i$	Var	$\bar{t}_i$	Var	$\bar{t}_i$	Var
2	.05266	.00329	.24945	.03055	.17978	.01820
3	.11135	.00743	.39813	.03104	.30206	.02250
4	.18111	.01295	.51361	.02756	.40442	.02376
5	.26278	.01996	.60965	.02395	.50000	.02404
6	.36378	.02862	.69689	.01951	.59558	.02376
7	.49258	.03901	.78335	.01551	.69794	.02250
8	.67419	.04471	.87648	.01087	.82022	.01820

Table IIX. Expected values and variances of the pseudo-standardized order statistics t_i; $N = 10$; $\alpha = 1.0; 0.9; 0.7; 0.5; 0.3; 0.1; 0.01; 0$

i	$\alpha = 1.0$		$\alpha = 0.9$		$\alpha = 0.7$		$\alpha = 0.5$	
	$\bar{t}_i$	Var	$\bar{t}_i$	Var	t_i	Var	t_i	Var
2	.04513	.00236	.05450	.00311	.07762	.00498	.11300	.00842
3	.09433	.00516	.11179	.00656	.16086	.00900	.20630	.01299
4	.14993	.00919	.17218	.01053	.22466	.01315	.29037	.01595
5	.21583	.01430	.24220	.01556	.30178	.01719	.37174	.01877
6	.29320	.02108	.32255	.02166	.38580	.02159	.45824	.02153
7	.38896	.02890	.41902	.02862	.48212	.02632	.55039	02426
8	.51096	.03779	.53780	.03548	.59808	.03106	.65797	.02532
9	.68753	.04271	.70796	.03931	.74950	.03050	.79131	.02274

i	$\alpha = 0.3$		$\alpha = 0.1$		$\alpha = 0.01$		$\alpha = 0$	
	$\bar{t}_i$	Var	$\bar{t}_i$	Var	$\bar{t}_i$	Var	$\bar{t}_i$	Var
2	.15842	.01412	.21145	.02299	.23801	.02858	.16950	.01644
3	.27057	.01819	.34623	.02572	.38079	.02969	.28152	.01991
4	.36657	.01977	.44842	.02465	.48652	.02706	.37400	.02114
5	.45373	.02083	.53573	.02273	.57491	.02352	.45866	.02126
6	.53780	.02169	.61704	.02081	.65296	.01984	.54134	.02126
7	.62475	.02175	.69597	.01819	.72863	.01648	.62600	.02114
8	.71953	.02064	.77754	.01546	.80454	.01299	.71848	.01991
9	.83236	.01167	.87145	.01126	.88718	.00937	.83050	.01644

Table IX. Expected values and variances of the pseudo-standardized order statistics t_i; $N = 19$; $\alpha = 1.0$; 0.01; 0.

i	$\alpha = 1.0$		$\alpha = 0.01$		$\alpha = 0$	
	$\bar{t}_i$	Var	$\bar{t}_i$	Var	$\bar{t}_i$	Var
2	.01741	.00034	.19243	.02070	.12018	.00903
3	.03586	.00077	.30063	.02210	.19555	.01099
4	.05559	.00131	.37619	.02069	.25387	.01161
5	.07659	.00200	.43654	.01927	.30288	.01176
6	.09916	.00280	.48685	.01779	.34724	.01177
7	.12323	.00374	.53083	.01625	.38802	.01190
8	.14969	.00488	.56992	.01487	.42624	.01180
9	.17844	.00631	.60660	.01371	.46348	.01179
10	.20919	.00789	.64128	.01250	.50000	.01170
11	.24344	.00989	.67381	.01136	.53652	.01179
12	.28213	.01218	.70617	.01028	.57376	.01180
13	.32589	.01503	.73837	.00931	.61198	.01190
14	.37627	.01820	.77077	.00847	.65276	.01177
15	.43659	.02229	.80399	.00773	.69712	.01176
16	.51001	.02718	.84061	.00688	.74613	.01161
17	.60645	.03168	.88037	.00592	.80445	.01099
18	.74559	.03252	.92848	.00456	.87982	.00903

Table X. Expected values and variances of the pseudo-standardized order statistics $\bar{t}_i$; $N = 20$; $\alpha = 1.0$; 0.9; 0.7; 0.5; 0.3; 0.1; 0.01; 0.

i	$\alpha = 1.0$		$\alpha = 0.9$		$\alpha = 0.7$		$\alpha = 0.5$	
	$\bar{t}_i$	Var	$\bar{t}_i$	Var	$\bar{t}_i$	Var	$\bar{t}_i$	Var
2	.01657	.00031	.02144	.00047	.03688	.00112	.06114	.00258
3	.03356	.00067	.04269	.00098	.06943	.00199	.11072	.00402
4	.05174	.00116	.06475	.00159	.10071	.00287	.15401	.00510
5	.07149	.00173	.08753	.00228	.13128	.00372	.19333	.00580
6	.09223	.00242	.11123	.00300	.16165	.00451	.23146	.00654
7	.11484	.00326	.13626	.00385	.19241	.00539	.26863	.00713
8	.13862	.00417	.16279	.00485	.22383	.00637	.30407	.00784
9	.16394	.00531	.19089	.00602	.25604	.00740	.33826	.00857
10	.19162	.00666	.22041	.00746	.28883	.00867	.37300	.00934
11	.22183	.00808	.25308	.00902	.32415	.00997	.40939	.01015
12	.25625	.00987	.28866	.01088	.36158	.01142	.44642	.01084
13	.29470	.01206	.32756	.01272	.40151	.01276	.48585	.01165
14	.33807	.01464	.37145	.01521	.44534	.01453	.52708	.01127
15	.38864	.01769	.42048	.01771	.49312	.01618	.57296	.01383
16	.44894	.02159	.47944	.02111	.54900	.01837	.62298	.01458
17	.52230	.02623	.54986	.02467	.61399	.02044	.68125	.01552
18	.61640	.03060	.64165	.02736	.69634	.02155	.75098	.01562
19	.75401	.03043	.77269	.02726	.81003	.02028	.84496	.01396

i	$\alpha = 0.3$		$\alpha = 0.1$		$\alpha = 0.01$		$\alpha = 0$	
	$\bar{t}_i$	Var	$\bar{t}_i$	Var	$\bar{t}_i$	Var	$\bar{t}_i$	Var
2	.10205	.00631	.15822	.01476	.18924	.02082	.11640	.00852
3	.17127	.00825	.25307	.01666	.29327	.02194	.19015	.01038
4	.22658	.00885	.32267	.01629	.36775	.02093	.24688	.01096
5	.27627	.00932	.38093	.01514	.42682	.01908	.29483	.01108
6	.32087	.00963	.42848	.01409	.47613	.01747	.33733	.01116
7	.36109	.00975	.47121	.01328	.51924	.01589	.37568	.01114
8	.39963	.00990	.51045	.01249	.55798	.01473	.41239	.01123
9	.43717	.01007	.54686	.01161	.59324	.01331	.44792	.01120
10	.47240	.01012	.58142	.01081	.62715	.01202	.48280	.01128
11	.50803	.01020	.61434	.01020	.65925	.01089	.51720	.01128
12	.54411	.01034	.64687	.00972	.69041	.00990	.56208	.01120
13	.58080	.01046	.67938	.00901	.72020	.00898	.58761	.01123
14	.61855	.01062	.71246	.00847	.75038	.00810	.62432	.01114
15	.66014	.01060	.74567	.00798	.78156	.00725	.66267	.01116
16	.70328	.01049	.78153	.00743	.81242	.00643	.70517	.01108
17	.75168	.01037	.82060	.00676	.84622	.00582	.75312	.01096
18	.80881	.00995	.86396	.00605	.88368	.00509	.80985	.01038
19	.88298	.00852	.91794	.00468	.93025	.00388	.88360	.00852

Table XI. Expected values and variances of the pseudo-standardized order statistics t_i; $N = 30$; $\alpha = 1.0;\ 0.9;\ 0.7;\ 0.5;\ 0.3;\ 0.1;\ 0.01;\ 0$.

i	$\alpha = 1.0$		$\alpha = 0.9$		$\alpha = 0.7$		$\alpha = 0.5$	
	$\bar{t}_i$	Var	$\bar{t}_i$	Var	$\bar{t}_i$	Var	$\bar{t}_i$	Var
2	.00952	.00010	.01302	.00017	.02525	.00052	.04635	.00158
3	.01948	.00023	.02562	.00035	.04737	.00094	.08286	.00232
4	.02971	.00037	.03811	.00053	.06767	.00130	.11322	.00285
5	.04030	.00055	.05087	.00072	.08600	.00161	.14125	.00327
6	.05156	.00075	.06343	.00095	.10482	.00192	.16685	.00360
7	.06282	.00097	.07680	.00120	.12373	.00226	.19172	.00400
8	.07475	.00123	.09053	.00148	.14152	.00258	.21528	.00431
9	.08744	.00154	.10499	.00181	.16006	.00299	.23753	.00455
10	.10038	.00187	.11981	.00217	.17860	.00332	.25986	.00483
11	.11416	.00226	.13506	.00258	.19763	.00374	.28171	.00519
12	.12848	.00268	.15058	.00300	.21644	.00411	.30312	.00546
13	.14353	.00313	.16758	.00348	.23547	.00463	.32524	.00580
14	.15940	.00368	.18472	.00402	.25516	.00513	.34748	.00620
15	.17655	.00425	.20330	.00468	.27569	.00557	.36940	.00658
16	.19482	.00491	.22247	.00544	.29688	.00614	.39104	.00695
17	.21465	.00570	.24314	.00622	.31887	.00684	.41371	.00739
18	.23526	.00658	.26482	.00700	.34192	.00754	.43633	.00784
19	.25765	.00751	.28837	.00802	.36616	.00821	.46077	.00836
20	.28142	.00854	.31408	.00913	.39216	.00900	.48609	.00868
21	.30904	.00982	.34144	.01038	.41926	.00985	.51260	.00915
22	.33796	.01115	.37174	.01180	.44896	.01083	.54085	.00967
23	.37105	.01285	.40464	.01328	.48144	.01201	.57110	.01030
24	.40872	.01473	.44265	.01516	.51684	.01312	.60376	.01090
25	.45199	.01703	.48547	.01727	.55830	.01426	.64041	.01144
26	.50445	.01969	.53626	.01975	.60349	.01574	.68158	.01196
27	.57100	.02316	.59684	.02197	.65857	.01722	.72914	.01244
28	.65552	.02617	.67739	.02375	.72976	.01780	.78670	.01243
29	.77763	.02592	.79053	.02267	.82820	.01612	.86430	.01111

Table XI. (Continued)

i	$\alpha = 0.3$		$\alpha = 0.1$		$\alpha = 0.01$		$\alpha = 0$	
	$\bar{t}_i$	Var	$\bar{t}_i$	Var	$\bar{t}_i$	Var	$\bar{t}_i$	Var
2	.08178	.00415	.13719	.01153	.16807	.01660	.09672	.00602
3	.13819	.00545	.21867	.01318	.26007	.01819	.15742	.00748
4	.18235	.00613	.27671	.01311	.32612	.01748	.20276	.00778
5	.21962	.00632	.32399	.01261	.37764	.01632	.24049	.00804
6	.25206	.00641	.36325	.01198	.41920	.01508	.27329	.00800
7	.28263	.00653	.39692	.01128	.45566	.01390	.30315	.00796
8	.31112	.00660	.42751	.01082	.48730	.01263	.33041	.00798
9	.33689	.00669	.45629	.01027	.51517	.01188	.35607	.00791
10	.36210	.00676	.48288	.00977	.54114	.01106	.37993	.00786
11	.38589	.00672	.50733	.00930	.56542	.01026	.40314	.00779
12	.40882	.00676	.53074	.00877	.58819	.00959	.42524	.00782
13	.43158	.00674	.55332	.00839	.60984	.00892	.44692	.00786
14	.45401	.00687	.57419	.00797	.62951	.00831	.46800	.00788
15	.47674	.00680	.59504	.00769	.64903	.00774	.48938	.00788
16	.49858	.00692	.61558	.00736	.66814	.00735	.51062	.00788
17	.52059	.00714	.63551	.00707	.68706	.00691	.53200	.00788
18	.54353	.00731	.65546	.00688	.70533	.00659	.55308	.00786
19	.56606	.00739	.67469	.00663	.72323	.00620	.57476	.00782
20	.58926	.00757	.69490	.00639	.74154	.00579	.59686	.00779
21	.61338	.00754	.71473	.00618	.75951	.00545	.62007	.00786
22	.63848	.00762	.73512	.00596	.77792	.00505	.64393	.00791
23	.66444	.00770	.75611	.00572	.79643	.00465	.66959	.00798
24	.69250	.00778	.77840	.00560	.81585	.00437	.69685	.00796
25	.72234	.00778	.80211	.00538	.83632	.00407	.72671	.00800
26	.75707	.00782	.82682	.00506	.85851	.00369	.75951	.00804
27	.79534	.00772	.85549	.00469	.88341	.00333	.79724	.00778
28	.84171	.00749	.88847	.00430	.91079	.00292	.84258	.00748
29	.90180	.00624	.93116	.00335	.94569	.00224	.90328	.00602

Table XII. Test level matrix of [STOO(2,9,10) ++ STOO(2,5,10)]

$T_1 \setminus T_2$	.005	.010	.015	.020	.025	.030	.035	.040	.045	.050	.055	Q_1
.050	82.3	-	-	-	-	-	-	-	-	-	-	87.1
.010	100.0	54.2	-	-	-	-	-	-	-	-	-	63.2
.015	61.4	69.4	31.8	-	-	-	-	-	-	-	-	45.7
.020	39.8	47.9	43.8	18.1	-	-	-	-	-	-	-	33.6
.025	24.8	34.1	36.7	29.6	11.7	-	-	-	-	-	-	25.0
.030	17.2	23.4	26.3	27.9	21.1	5.7	-	-	-	-	-	18.8
.035	9.2	15.0	22.1	20.1	19.1	13.7	3.6	-	-	-	-	14.2
.040	6.1	12.2	12.7	16.4	16.4	11.7	9.2	2.4	-	-	-	10.8
.045	2.4	6.1	9.5	13.2	14.4	10.3	9.2	5.7	-	-	-	8.7
.050	-	3.7	7.8	9.9	8.3	10.7	7.8	5.4	2.6	-	-	6.2
.055	-	3.2	5.4	8.0	7.2	6.5	5.4	6.7	4.8	-	-	4.7
.060	-	-	3.6	6.5	4.2	7.2	4.2	4.5	4.2	3.2	-	3.6
.065	-	-	3.2	4.5	3.2	4.5	4.6	5.4	4.2	3.6	-	2.6
.070	-	-	-	3.2	2.4	2.1	2.4	3.6	-	3.6	2.6	2.0
.075	-	-	-	3.2	3.2	2.1	-	-	-	-	-	1.6
.080	-	-	-	-	-	-	-	2.6	-	-	-	1.3
Q_2	53.3	32.8	20.0	12.4	7.5	4.6	3.0	1.8	1.3	0.9	0.7	-

Empty classes indicate a TL < 2%

Table XIII. Percentiles of the pseudo-standardized order statistics t_i; $N = 9$; $\alpha = 1.0$; 0.01; 0.

i	P = 5%			P = 50%			P = 95%		
	1.0	0.01	0	1.0	0.01	0	1.0	0.01	0
2	.00360	.01925	.01356	.03762	.22211	.15266	.16552	.57517	.43458
3	.01058	.11315	.07697	.08978	.39537	.28936	.28049	.69314	.55789
4	.04021	.23032	.15567	.15591	.52107	.39782	.40042	.77153	.66480
5	.07573	.33876	.24546	.23636	.62338	.49797	.53579	.83869	.75507
6	.12580	.44881	.33115	.33993	.71242	.59920	.68499	.89646	.83837
7	.19057	.55779	.43024	.48060	.80267	.70835	.83628	.95218	.92053
8	.30289	.68261	.56332	.69461	.90069	.84544	.97188	.99117	.98628

Table XIV. Percentiles of the pseudo-standardized order statistics t_i; N = 10; α = 1.0;0.9;0.7;0.5; 0.3;0.1;0.01;0

P = 5%

i	1.0	0.9	0.7	0.5	0.3	0.1	0.01	0
2	.00330	.0032	.0050	.0080	.0124	.0160	.01884	.01316
3	.00854	.0200	.0304	.0483	.0688	.0954	.10728	.07351
4	.02726	.0439	.0700	.1037	.1439	.1855	.20797	.14793
5	.06238	.0759	.1159	.1685	.2222	.2771	.30877	.22203
6	.09652	.1155	.1702	.2302	.2988	.3660	.40639	.30031
7	.14620	.1709	.2339	.3021	.3802	.4611	.50183	.38278
8	.21093	.2370	.3137	.3872	.4686	.5571	.60108	.46876
9	.31968	.3471	.4307	.5146	.5916	.6727	.70971	.58974

P = 50%

i	1.0	0.9	0.7	0.5	0.3	0.1	0.01	0
2	.03440	.03607	.05765	.09118	.13688	.18669	.21031	.14563
3	.07749	.09184	.13166	.19076	.25976	.33991	.37558	.27395
4	.12972	.15152	.20720	.27966	.36033	.45410	.49266	.37214
5	.19244	.22042	.28585	.36127	.45293	.54161	.58666	.45957
6	.26969	.30178	.37105	.45065	.53815	.63071	.66546	.54606
7	.36629	.39852	.47380	.54923	.63129	.71289	.74186	.63466
8	.49981	.55147	.59860	.66721	.73311	.79551	.82291	.73343
9	.71135	.73320	.77489	.81936	.85852	.89651	.91026	.85707

P = 95%

i	1.0	0.9	0.7	0.5	0.3	0.1	0.01	0
2	.14050	.1630	.2193	.2917	.3907	.5019	.56058	.41484
3	.23410	.2697	.3360	.4185	.5122	.6162	.67353	.53036
4	.33618	.3712	.4416	.5169	.6095	.7024	.74712	.62008
5	.44267	.4787	.5415	.6150	.6969	.7613	.80582	.70133
6	.56985	.5965	.6507	.7128	.7780	.8353	.85880	.77687
7	.70420	.7302	.7636	.8075	.8562	.8926	.91000	.85549
8	.84963	.8523	.8847	.9071	.9314	.9484	.95781	.93009
9	.97339	.9749	.9811	.9831	.9878	.9907	.99209	.98746

Table XV. Percentiles of the pseudo-standardized order statistics t_i; $N = 19$; $\alpha = 1.0; 0.01; 0$

i	P = 5%			P = 50%			P = 95%		
	1.0	0.01	0	1.0	0.01	0	1.0	0.01	0
2	.00218	.01424	.00886	.02313	.16477	.09589	.05566	.46935	.29586
3	.00306	.07502	.04211	.03097	.29079	.18379	.08891	.56328	.38491
4	.00509	.14488	.08405	.04744	.37304	.24747	.12529	.61700	.43793
5	.00957	.20647	.12707	.06823	.43808	.29859	.16137	.66168	.48764
6	.01768	.26232	.16298	.09020	.49060	.34520	.20056	.69930	.52891
7	.02648	.31500	.21117	.11282	.53730	.38543	.23895	.73041	.57097
8	.03529	.36510	.24335	.13810	.57762	.42472	.28192	.75689	.60504
9	.06986	.40977	.28169	.16644	.61488	.46227	.32877	.78380	.64536
10	.08853	.45236	.32090	.19544	.65005	.50047	.37404	.80745	.67730
11	.10630	.49126	.35328	.22940	.68262	.53986	.42960	.83030	.71410
12	.12692	.53204	.39482	.26755	.71494	.57722	.48766	.85293	.75168
13	.15054	.57062	.42157	.31070	.74742	.61131	.55150	.87521	.78937
14	.17780	.61185	.46155	.36166	.78059	.65319	.61908	.89820	.82762
15	.21278	.65280	.49943	.42371	.81421	.70105	.70090	.92370	.86644
16	.25512	.69528	.55019	.50117	.85171	.75323	.79365	.95130	.90818
17	.31565	.74908	.60739	.60898	.89329	.81665	.89040	.98233	.95500
18	.41388	.81173	.68936	.77525	.94459	.89891	.98029	.99478	.98675

Table XVI. Percentiles of the pseudo-standardized order statistics t_i; $N = 20$; $\alpha = 1.0; 0.01; 0$

i	P = 5%			P = 50%			P = 95%		
	1.0	0.01	0	1.0	0.01	0	1.0	0.01	0
2	.0016	.0129	.0350	.0159	.1606	.0955	.0503	.4706	.2969
3	.0027	.0698	.0670	.0275	.2822	.1791	.0873	.5502	.3783
4	.0056	.1339	.1000	.0447	.3655	.2389	.1191	.6099	.4326
5	.0148	.1994	.1350	.0641	.4267	.2893	.1491	.6558	.4758
6	.0286	.2566	.1683	.0843	.4784	.3331	.1891	.6904	.5195
7	.0378	.3036	.2056	.1050	.5241	.3713	.2222	.7196	.5544
8	.0519	.3468	.2389	.1284	.5642	.4104	.2607	.7459	.5914
9	.0638	.3915	.2755	.1528	.6013	.4463	.2989	.7703	.6238
10	.0792	.4345	.3073	.1804	.6351	.4819	.3432	.7932	.6578
11	.0951	.4750	.3388	.2106	.6671	.5175	.3908	.8167	.6897
12	.1137	.5154	.3755	.2456	.6997	.5536	.4396	.8394	.7239
13	.1334	.5561	.4101	.2835	.7282	.5895	.4987	.8592	.7612
14	.1570	.5910	.4476	.3272	.7591	.6259	.5564	.8839	.7964
15	.1893	.6264	.4785	.3783	.7906	.6662	.6250	.9048	.8300
16	.2253	.6663	.5250	.4394	.8212	.7109	.7098	.9278	.8680
17	.2689	.7068	.5714	.5156	.8566	.7614	.7957	.9516	.9120
18	.3244	.7525	.6266	.6193	.8958	.8226	.8968	.9744	.9559
19	.4234	.8108	.7086	.7854	.9460	.9053	.9797	.9952	.9912

Table XVII. Sampling distributions of the pseudo-standardized order statistics t_i; N = 10; Weibull dbn, $\alpha = 1.0$

t \ i	2	3	4	5	6	7	8	9
0.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.02	29.280	11.924	3.608	0.745	0.141	0.005	0.000	0.000
0.04	56.129	24.611	7.974	1.780	0.326	0.009	0.000	0.000
0.06	73.750	38.351	15.305	4.525	1.091	0.181	0.006	0.000
0.08	83.686	51.483	24.956	9.156	2.740	0.637	0.066	0.000
0.10	89.241	62.805	35.379	15.245	5.345	1.429	0.248	0.016
0.12	92.663	71.983	45.418	22.316	8.807	2.639	0.596	0.058
0.14	94.956	79.126	54.560	29.972	13.036	4.352	1.124	0.142
0.16	96.498	84.496	62.652	37.802	18.037	6.624	1.869	0.304
0.18	97.540	88.472	69.680	45.441	23.727	9.449	2.887	0.561
0.20	98.255	91.459	75.612	52.636	29.811	12.761	4.183	0.907
0.24	99.105	95.437	84.471	64.888	41.784	20.577	7.554	1.815
0.28	99.593	97.562	90.241	74.318	52.735	29.365	12.071	3.103
0.32	99.845	98.651	93.915	81.876	62.765	39.087	17.994	5.007
0.36	99.928	99.259	96.219	87.543	71.385	48.576	24.777	7.456
0.40	99.963	99.614	97.697	91.836	78.474	57.271	31.372	10.472
0.44	99.986	99.816	98.611	94.860	84.179	64.814	38.329	14.037
0.48	99.998	99.935	99.182	96.560	88.484	71.660	46.024	18.247
0.52	100.000	99.986	99.563	97.804	91.779	77.818	53.866	22.798
0.56	-	99.990	99.765	98.672	94.489	83.097	60.873	27.695
0.60	-	99.990	99.887	99.227	96.231	87.338	67.503	32.902
0.64	-	99.990	99.963	99.574	97.481	90.679	73.439	38.652
0.68	-	99.991	99.979	99.823	98.562	93.514	78.570	44.969
0.72	-	99.997	99.995	99.919	99.232	95.845	83.536	51.385
0.76	-	100.00	100.000	99.971	99.634	97.515	87.861	57.851
0.80	-	-	-	99.998	99.848	98.572	91.401	64.710
0.82	-	-	-	100.000	99.905	98.951	92.962	68.025
0.84	-	-	-	-	99.948	99.264	94.370	71.285
0.86	-	-	-	-	99.976	99.497	95.650	74.641
0.88	-	-	-	-	99.992	99.667	96.823	78.120
0.90	-	-	-	-	99.999	99.800	97.853	81.690
0.92	-	-	-	-	100.000	99.899	98.668	85.278
0.94	-	-	-	-	-	99.960	99.238	88.856
0.96	-	-	-	-	-	99.987	99.591	92.502
0.98	-	-	-	-	-	99.995	99.805	96.238
1.00	-	-	-	-	-	100.00	100.00	100.000

Table XIIX. Sampling distributions of the pseudo-standardized order statistics t_i; N = 10; Weibull dbn, $\alpha = 0.01$

t \ i	2	3	4	5	6	7	8	9
0.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.02	5.308	0.385	0.012	0.004	0.000	0.000	0.000	0.000
0.04	10.609	0.838	0.027	0.012	0.000	0.000	0.000	0.000
0.06	15.753	1.617	0.097	0.026	0.000	0.000	0.000	0.000
0.08	20.768	2.772	0.256	0.037	0.000	0.000	0.000	0.000
0.10	25.771	4.334	0.527	0.057	0.003	0.000	0.000	0.000
0.12	30.644	6.252	0.949	0.120	0.009	0.000	0.000	0.000
0.14	35.181	8.458	1.562	0.249	0.017	0.000	0.000	0.000
0.16	39.449	10.953	2.358	0.427	0.028	0.000	0.000	0.000
0.18	43.656	13.691	3.315	0.637	0.052	0.004	0.000	0.000
0.20	47.858	16.638	4.470	0.904	0.101	0.007	0.000	0.000
0.24	55.894	23.399	7.565	1.774	0.295	0.035	0.000	0.000
0.28	63.373	30.642	11.721	3.393	0.693	0.127	0.010	0.000
0.32	70.402	38.450	16.796	5.743	1.580	0.307	0.036	0.000
0.36	76.384	46.763	23.169	9.028	2.877	0.621	0.059	0.000
0.40	81.668	54.980	30.804	13.353	4.620	1.219	0.165	0.004
0.44	85.914	62.989	38.850	19.428	7.474	2.174	0.383	0.047
0.48	89.402	70.512	47.168	26.490	11.444	3.743	0.798	0.113
0.52	92.507	77.302	56.073	34.342	17.096	6.281	1.453	0.237
0.56	94.968	83.330	64.775	43.350	24.284	10.082	2.748	0.468
0.60	96.929	88.507	73.322	53.427	32.677	15.395	4.930	0.974
0.64	98.363	92.294	80.933	63.709	42.906	22.505	8.208	1.731
0.68	99.219	95.440	87.361	73.253	54.133	31.740	13.332	3.145
0.72	99.644	97.552	92.421	81.712	65.412	43.377	21.084	5.838
0.76	99.862	98.903	96.000	89.089	75.802	55.508	30.954	10.215
0.80	99.958	99.603	98.301	94.399	85.236	67.824	42.471	16.551
0.82	99.980	99.789	99.027	96.285	89.181	73.898	48.984	20.674
0.84	99.994	99.910	99.495	97.672	92.464	79.669	56.161	25.495
0.86	100.000	99.974	99.762	98.614	95.093	84.959	63.774	31.154
0.88	-	99.997	99.901	99.243	97.079	89.614	71.358	37.835
0.90	-	100.000	99.969	99.657	98.455	93.433	78.577	45.623
0.92	-	-	99.996	99.888	99.318	96.306	85.289	54.367
0.94	-	-	100.000	99.981	99.783	98.250	91.109	64.051
0.96	-	-	-	99.998	99.961	99.285	95.244	75.113
0.98	-	-	-	99.998	99.984	99.688	97.809	87.413
1.00	-	-	-	100.00	100.00	100.00	100.00	100.000

Table XIX. Sampling distributions of the pseudo-standardized order statistics t_i; N = 10; normal distribution

t \ i	2	3	4	5	6	7	8	9
0.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.02	7.601	0.715	0.051	0.000	0.000	0.000	0.000	0.000
0.04	15.203	1.605	0.107	0.000	0.000	0.000	0.000	0.000
0.06	22.551	3.336	0.326	0.009	0.000	0.000	0.000	0.000
0.08	29.464	5.847	0.800	0.049	0.000	0.000	0.000	0.000
0.10	36.024	8.979	1.580	0.164	0.000	0.000	0.000	0.000
0.12	42.361	12.664	2.733	0.395	0.023	0.007	0.000	0.000
0.14	48.385	16.752	4.283	0.770	0.112	0.008	0.000	0.000
0.16	53.996	21.130	6.224	1.328	0.262	0.026	0.000	0.000
0.18	59.243	25.788	8.651	2.144	0.451	0.076	0.001	0.000
0.20	64.160	30.685	11.652	3.294	0.680	0.157	0.009	0.000
0.24	72.894	40.966	18.762	6.721	1.585	0.366	0.046	0.001
0.28	80.270	51.605	26.811	11.505	3.595	0.824	0.111	0.018
0.32	86.242	61.850	36.452	17.548	6.601	1.885	0.331	0.076
0.36	90.751	70.874	46.882	25.511	10.962	3.524	0.761	0.148
0.40	94.019	78.850	57.046	34.906	16.993	6.436	1.683	0.244
0.44	96.359	85.573	66.973	45.066	24.706	10.731	3.393	0.567
0.48	97.883	90.601	75.685	55.105	33.803	16.126	5.742	1.209
0.52	98.893	94.225	82.915	65.073	43.461	23.035	9.021	2.118
0.56	99.484	96.832	89.074	74.470	53.558	31.747	13.536	3.547
0.60	99.772	98.370	93.445	82.362	63.921	41.151	19.837	5.602
0.64	99.912	99.200	96.271	88.513	73.385	51.373	27,701	8.661
0.68	99.961	99.657	98.159	93.128	81.637	61.479	36.481	12.781
0.72	99.982	99.885	99.188	96.315	88.466	71.038	46.545	18.442
0.76	99.993	99.968	99.691	98.291	93.401	80.123	56.891	25.605
0.80	100.000	99.987	99.908	99.339	96.657	87.689	67.541	34.693
0.82	-	99.993	99.951	99.624	97.724	90.742	72.815	39.911
0.84	-	99.998	99.972	99.800	98.523	93.308	77.814	45.294
0.86	-	100.000	99.983	99.901	99.100	95.417	82.411	50.825
0.88	-	-	99.992	99.955	99.496	97.057	86.575	56.683
0.90	-	-	99.998	99.982	99.752	98.247	90.307	62.944
0.92	-	-	100.000	99.994	99.903	99.064	93.560	69.585
0.94	-	-	-	99.999	99.977	99.580	96.178	76.620
0.96	-	-	-	100.00	100.000	99.838	97.949	84.136
0.98	-	-	-	-	-	99.929	99.041	92.035
1.00	-	-	-	-	-	100.00	100.00	100.000

Table XX. Sampling distributions of the pseudo-standardized variable t; $N = 6$; $\alpha = 1.0; 0.9; 0.7; 0.5; 0.3; 0.1; 0$.

t	1.0	0.9	0.7	0.5	0.3	0.1	0
.125	27.83	24.36	18.13	13.24	9.64	7.20	9.28
.250	48.19	44.51	36.67	28.69	21.46	16.20	20.97
.375	63.20	60.40	53.12	44.76	35.60	27.61	34.82
.500	74.88	72.36	66.90	59.91	50.98	41.20	50.00
.625	83.41	81.88	78.31	73.07	66.17	57.00	65.18
.750	90.03	89.23	87.13	84.18	79.82	73.17	79.03
.875	95.39	95.06	94.09	92.65	90.80	87.82	90.72

Table XXI. Sampling distributions of the pseudo-standardized variable t; $N = 10$; $\alpha = 1.0; 0.9; 0.7; 0.5; 0.3; 0.1; 0$

t	1.0	0.9	0.7	0.5	0.3	0.1	0
.090	24.11	-	-	-	-	3.93	5.34
.100	26.39	-	-	-	-	4.42	6.03
.125	31.77	27.57	19.41	12.87	8.41	5.72	7.93
.225	40.57	-	-	-	-	11.96	16.52
.250	53.28	49.03	39.41	29.24	20.15	13.83	19.15
.300	60.10	-	-	-	-	17.80	24.60
.375	68.52	65.29	56.82	46.39	34.80	24.68	33.59
.450	75.31	-	-	-	-	32.67	43.27
.500	79.25	76.88	70.90	62.54	51.15	38.73	50.00
.625	86.83	85.39	81.65	75.88	67.47	55.55	66.40
.700	90.49	-	-	-	-	66.32	75.40
.750	92.48	91.72	89.61	86.47	81.44	73.16	80.85
.875	96.72	96.43	95.51	94.28	92.27	88.78	92.07

Table XXII. Sampling distributions of the pseudo-standardized variable t; N = 20; α = 1.0;0.9;0.7;0.5;0.3;0.1; 0.

t	1.0	0.9	0.7	0.5	0.3	0.1	0
.125	36.73	31.43	20.91	12.32	6.76	3.99	6.14
.250	59.58	54.64	43.07	29.88	18.20	10.57	16.59
.375	74.34	70.85	61.60	48.66	33.66	20.68	31.70
.500	83.95	81.75	75.58	65.89	51.50	35.07	50.00
.625	90.39	89.10	85.49	79.52	69.24	53.46	68.30
.750	94.81	94.17	92.32	89.24	83.74	73.37	83.41
.875	97.84	97.61	96.96	95.84	93.90	90.01	93.86

Table XXIII. Percentiles of the pseudo-standardized variable t; N = 10; α = 1.0; 0.1 and 0

P %	α = 1.0	α = 0.1	α = 0
12.5	.0435	.2323	.1810
25.0	.0938	.3780	.3035
37.5	.1542	.4904	.4055
50.0	.2280	.5853	.5000
62.5	.3197	.6732	.5945
75.0	.4461	.7635	.6965
87.5	.6377	.8635	.8190

Table XXIV. Analysis of a fatigue-test sample (Item 44, Ref.3)

i	x_i	t_i	TI	$t_i - t_{oi}$	$\log x_i$	t_i	TI	$t_i - t_{oi}$
1	70	-	-	-	4.8451	-	-	-
2	76	.0018	<	-.1677	4.8808	.0213	<	-.1482
3	80	.0031	<	-.2784	4.9031	.0346	<	-.2469
4	81	.0034	<	-.3706	4.9085	.0378	<	-.3662
5	86	.0049	<	-.4538	4.9345	.0533	<	-.4054
6	108	.0117	<	-.5296	5.0334	.1124	<	-.4289
7	142	.0222	<	-.6038	5.1523	.1833	<	-.4427
8	144	.0228	<	-.6957	5.1584	.1870	<	-.5315
9	282	.0653	<	-.7652	5.4502	.3611	<	-.4694
10	3318	-	-	-	6.5209	-	-	-

TX(2,5,10) = 0.0033; Q = 98.2%	TX(2,5,10) = 0.0367; Q = 99.4%
TX(2,9,10) = 0.0169; Q = 99.8%	TX(2,9,10) = 0.1238; Q = 99.8%
Bivariate TL = 2%	Bivariate TL = 2%
STOX(2,5,10) = 0.4490;Q = 0.9%	STOX(2,5,10) = 0.3814;Q 0.7 %
STOX(2,9,10) = 2.1636;Q = 1.3%	STOX(2,9,10) = 1.2641;Q 1.3%
Bivariate TL = 2%	Bivariate TL 2%
VJX = 8000 ; TL = 0.1%	VJX = 7100 ; TL = 0.3%

Table XXV. Analysis of a fatigue-test sample (Item 46, Ref.3)

i	x_i	t_i	TL	$t_i - t_{oi}$	$\log x_i$	t_i	TL	$t_i - t_{oi}$
1	7.1	-	-	-	3.8513	-	-	-
2	7.4	.1071	a	-.0624	3.8692	.1240	a	-.0455
3	7.8	.2500	a	-.0315	3.8921	.2827	a	+.0012
4	8.0	.3214	a	-.0526	3.9031	.3590	a	-.0116
5	8.4	.4643	a	+.0056	3.9243	.5059	a	+.0472
6	8.7	.5714	a	+.0301	3.9395	.6112	a	+.0699
7	9.0	.6786	a	+.0526	3.9542	.7131	a	+.0871
8	9.4	.8214	a	+.1029	3.9731	.8441	a	+.1256
9	9.6	.8928	a	-.0623	3.9823	.9078	a	+.0773
10	9.9	-	-	-	3.9956	-	-	-
TX(2,5,10) = 0.2857 ; Q > 52.6% TX(2.9.10) = 0.5134 ; Q > 41.5% Bivariate TL > 49.1% STOX(2,5,10) = 0.00768;Q>32.8% STOX(2,9,10) = 0.02583;Q>25.0% Bivariate TL>24.8% VJX = 1214 ; TL = 80.8%					TX(2,5,10) = 0.3179 ; Q > 45.3% TX(2,9,10) = 0.5435 ; Q > 37.7% Bivariate TL > 30.5% STOX(2,5,10) = 0.00443;Q>53.3% STOX(2.9.10) = 0.03866;Q>10.8% Bivariate TL > 6.1% VJX = 1205 ; TL = 47.1%			

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13. ABSTRACT

The concept of pseudo-standardized variable is explained and the fundamental properties of this variable are indicated. Its most important property of being scale and location invariant makes it useful as elements of shape operators, and its space being equal to the closed interval (0,1) has practical advantages.

Four types of shape operators are defined and examined. Twenty-five tables which simplify their practical applications have been prepared and are presented. Two examples concerning data of rotating beam fatigue performance illustrate the different numerical procedures.

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